

1. INTRODUCTION TO ANALYTICAL TOOLS

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EPH 1100: Analytical Tools

Reading: Hoy et al., Ch. 1.1–2.2.

- **Ch. 1:** Vocabulary of economic models (endogenous/exogenous variables, parameters, equilibrium conditions). Read carefully as reinforcement of today's notes.
- **§2.1–2.2:** Sets and number systems. Skim for familiarity with notation; refer back as needed. No need to memorize proofs.

1. Why Do Economists Use Math?

Economists have relied on mathematics for well over a century. Léon Walras, writing in the 1870s, showed that understanding how prices form across many interconnected markets can be stated as a problem of solving systems of equations. Alfred Marshall, whose *Principles of Economics* (1890) shaped the field forever, built every conclusion on algebra and geometry, even when he moved the math to appendices to keep the main text readable.¹

Why did these authors rely on math rather than verbal reasoning? Verbal reasoning is powerful. It conveys intuition, builds narrative, and reaches broad audiences. And for many economic questions, it does the job perfectly well. Consider a firm's total cost function: $C = wL + rK$. You could say the same thing in plain English with no loss of precision: “total cost is what the firm pays workers plus what it pays to rent capital.” No ambiguity and no information lost.

But consider a harder question: how does a firm's output depend on the labor and capital it employs? You might say “output increases when you hire more workers or acquire more machines.” True, but what else is true? Does it matter how much capital you already have when you add a worker? Do returns diminish as you add more of one input? Can labor and capital substitute freely for each other, or does production require them in fixed proportions? None of these questions has an obvious answer from the verbal statement alone. It would require many pages of verbal reasoning to convey all of those details, potentially leading to imprecision or ambiguity.

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¹The reliance on mathematics has only deepened over time. Backhouse (1998) finds that the proportion of articles using algebra in the *Economic Journal* and the *American Economic Review*—two of the leading journals in the profession—jumped from no more than 10 percent in 1930 to around 75 percent by 1980. Today, an article without mathematics would be quite a notable exception in either of these journals.

A production function that relates how different units of capital (K) and labor (L) can be combined to produce some level of output (Q), such as $Q = K^\alpha L^{1-\alpha}$, solves these issues all at once: it specifies the functional form, makes the role of each input explicit, and pins down exactly how output responds to changes in either variable. From that structure it is easier to make causal connections and derive testable predictions. Even if not empirically testable, it helps to assess the logical validity of an argument. If you're interested in understanding why this is the "economic way of thinking", I highly recommend that you read [Lazear \(2000\)](#).²

2. The Language of Models

Economists and other social scientists think in terms of *models*: deliberate simplifications that strip a situation down to its key moving parts, then analyze those parts carefully. A model does not describe reality in full, and this choice is intentional. Think of a 1:1 map of Austin; we would be as lost if we didn't have any map at all. We want to simplify reality, isolating the forces that matter for a specific question. With maps, we may want to highlight the city grid and political boundaries for one question, or the elevation, waterways, and vegetation for others.

Every model has three kinds of ingredients:

- **Variables:** quantities that can take different values (e.g., prices, quantities, income, profit).
- **Parameters:** numbers that are fixed for a given problem (e.g., the slope of a demand curve, the preferences of consumers, how many votes a candidate needs to win the election). They describe the environment; they do not change within the model.
- **Equations:** the causal relationships between variables, usually from behavioral assumptions (how people respond to incentives) or equilibrium conditions (what has to be true in balance).

The most important distinction to internalize is between **endogenous** and **exogenous** variables:

Endogenous	Determined <i>inside</i> the model (what we are solving for).
Exogenous	Given from <i>outside</i> the model (taken as fixed).

Before doing any algebra, the first question to ask is: *what am I solving for, and what am I treating as given?* Getting this right is half the work.

²Whether we like it or not and whether it is fully warranted or not, a sociological reality: across the sciences, the use of mathematics has come to be associated with rigor and credibility. One can debate whether mathematical formalization always improves science, but in practice it shapes what gets published, what gets compared across studies, and what gets taken seriously. Thus, engaging with most of what gets published in economic sciences today requires one to be familiar with these tools.

3. Worked Example: Market Equilibrium

A firm sells widgets. Market demand is:

$$Q^D = 100 - 2P$$

where P is the price per unit. Supply is fixed at $\bar{Q} = 60$ units (say, the factory has a fixed capacity).

Question: What is the equilibrium price?

Step 1: Identify the moving parts.

- Endogenous: P (the price we are solving for).
- Exogenous: $\bar{Q} = 60$ (fixed supply), and the parameters 100 and 2 in the demand function.

Step 2: State the equilibrium condition.

In equilibrium, the market clears: quantity demanded equals quantity supplied.

$$Q^D = \bar{Q}$$

Step 3: Solve.

$$100 - 2P = 60 \implies 2P = 40 \implies P = 20$$

At a price of \$20, buyers want exactly 60 units, which is exactly what is available. The market clears.

Now suppose a drought destroys part of the supply, and $\bar{Q}$ falls to 50. What happens to the price?

$$100 - 2P = 50$$

$$2P = 50$$

$$P = 25$$

Price rises from \$20 to \$25. The direction we get from economic theory: with less supply, buyers compete more aggressively, bidding price up. The model gives us the exact magnitude of the increase, not just the direction.

Notice what we just did: we asked how the endogenous variable (P) responds to a change in an exogenous variable ($\bar{Q}$). This type of question is called **comparative statics**, and it is one of the central tools theoretical analysis in both economics and political science. We will build the machinery to answer it rigorously in the second half of the course.

4. Course Roadmap

Here is where we are going and why, in rough order:

1. **Derivatives:** How does a function change when its input changes? This is the fundamental tool for measuring marginal effects: what's the cost of producing one additional unit? How many votes do I get for each additional dollar spent in advertising?
2. **Optimization:** How do you find the best choice (the price that maximizes profit, the output that minimizes cost)? Derivatives are the main tool.
3. **Linear algebra:** How do you solve systems with multiple equations and multiple unknowns? This is the toolkit for systems of equations with many variables and data science (econometrics, statistics, machine learning).
4. **Multivariate optimization:** What changes when you are optimizing over two or more variables at once (like a consumer choosing quantities of two goods subject to a budget)?
5. **Comparative statics:** How does the optimal choice change when the environment changes (when campaign spending limits are tightened, when a tax is imposed, when a constraint is removed)?
6. **Integration:** How do you add up a continuous stream of small effects? Consumer surplus, total cost from marginal cost.

Each topic builds on the previous one. The thread running through all of it is the same: use math to answer a specific economic question precisely.

References

- Backhouse, R. E. (1998). If mathematics is informal, then perhaps we should accept that economics must be informal too. *The Economic Journal*, 108(451):1848–1858.
- Lazear, E. P. (2000). Economic imperialism. *The Quarterly Journal of Economics*, 115(1):99–146.